# Adaptive Neuro-Fuzzy Inference System based Fractal Image Compression

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*Abstract*—**This paper presents an Adaptive Neuro-Fuzzy Inference System (ANFIS) model for fractal image compression. One of the image compression techniques in the spatial domain is Fractal Image Compression (FIC) but the main drawback of FIC using traditional exhaustive search is that it involves more computational time due to global search. In order to improve the computational time and compression ratio, artificial intelligence technique like ANFIS has been used. Feature extraction reduces the dimensionality of the problem and enables the ANFIS network to be trained on an image separate from the test image thus reducing the computational time. Lowering the dimensionality of the problem reduces the computations required during the search. The main advantage of ANFIS network is that it can adapt itself from the training data and produce a fuzzy inference system. The network adapts itself according to the distribution of feature space observed during training. Computer simulations reveal that the network has been properly trained and the fuzzy system thus evolved, classifies the domains correctly with minimum deviation which helps in encoding the image using FIC.**

*Index Terms*—**ANFIS. FIC, Standard deviation, Skew, neural network**

## I. Introduction

Fractal Image compression method exploits similarities in different parts of the image. In this an image is represented by fractals rather than pixels, where each fractal is defined by unique Iterated Function System (IFS) consisting of a group of affine transformations. FIC employs a special type of IFS called as Partitioned Iterated Function System (PIFS). Collage Theorem is employed for PIFS and gray scale images which is equivalent to IFS for binary images. The collage theorem performs the encoding of gray scale images in an effective manner. The key point for fractal coding is to extract the fractals which are suitable for approximating the original image and these fractals are represented as set of affine transformations [1-3]. Fractal image coding introduced by Barnsley and Jacquin is the outcome of the study of the iterated function system developed in the last decade. Because of its high compression ratio and simple decompression method, many researchers have done a lot of research on it. But the main drawback of their work can be related to large computational time for image compression.

In order to reduce the number of computations required many sub-optimal techniques have been proposed. It is shown that the computational time can be improved by performing the search in a small sub-set of domain pool rather than over the whole space. Clustering purpose is to divide a given group of objects in a number of groups, in order that the objects in a particular cluster would be similar among the objects of the other ones [4]. In this method 'N' objects are divided into 'M' clusters where M<N, according to the minimization of some criteria. The problem is to classify a group of samples. These samples form clusters of points in a n-dimensional space. These clusters form groups of similar samples. Data clustering algorithms can be hierarchical. Hierarchical algorithms find successive clusters using previously established clusters. Hierarchical algorithms can be agglomerative ("bottom-up") or divisive ("top-down"). Agglomerative algorithms begin with each element as a separate cluster and merge them into successively larger clusters. Divisive algorithms begin with the whole set and proceed to divide it into successively smaller clusters.

A fuzzy system is the one which has the capability to estimate the output pattern by considering the input patterns based on the membership functions created. It works with the implementation of rules which are written based on the behaviour of the system considered. The main disadvantage of the fuzzy systems is the large computational time required in tuning the rules. An artificial neural network (ANN), often just called a "neural network" (NN), is a mathematical model or computational model based on biological neural networks. It consists of an interconnected group of artificial neurons and processes information using a connectionist approach to computation [5].

The main objective of this paper is to develop an a technique by incorporating the combined concept of fuzzy and neural network called as Adaptive Neuro-fuzzy Inference System. This technique incorporates the concept of NN in creating the rules and produces a fuzzy model based on Tagaki-Sugeno approach. This fuzzy inference system can be employed to classify the domain pool blocks of a gray level image, thus




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improving the encoding time. In view of this, Section II deals with the concept of FIC. The self and affine transformations along with the collage theorem are dealt with. Section III deals with the concept of ANFIS. Results and Discussions are explained in Section IV. Conclusions are drawn in Section V.

## II. FRACTAL IMAGE COMPRESSION

The fractal image compression algorithm is based on the fractal theory of self-similar and self-affine transformations

### *A. Self-Affine and Self-Similar Transformations*

In this section we present the basic theory involved in Fractal Image Compression. It is basically based on fractal theory of self-affine transformations and self-similar transformations. A self-affine transformation $W: R^n \rightarrow R^n$ is a transformation of the form $W(x)=T(x)+b$, where $T$ is a linear transformation on $R^n$ and $b \in R^n$ is a vector.

A mapping $W: D \rightarrow D,\ D \subseteq R^n$ is called a contraction on $D$ if there is a real number $c,\ 0<c<1$ such that $d(W(x), W(y)) \leq cd(x,y)$ for $x,y \in D$ and for a metric $d$ on .The real number $c$ is called the contractivity of $W$.

$d(W(x), W(y)) = cd(x,y)$ then $W$ is called a similarity.

A family $\{w_1, \ldots. w_m\}$ of contractions is known as Local Iterated function scheme (LIFS). If there is a subset $F \subseteq D$ such that for a LIFS $\{w_1, \ldots. w_m\}$

$$w_i(F) \tag{1}$$

Then $F$ is said to be invariant for that LIFS. If $F$ is invariant under a collection of similarities, $F$ is known as a self-similar set. Let $S$ denote the class of all non-empty compact subsets of $D$. The $\delta$ -parallel body of $A \in S$ is the set of points within distance $\delta$ of A, i.e.

$$A_\delta = \{x \in D: |x-a| \leq \delta, a \in A\} \tag{2}$$

Let us define the distance $d(A,B)$ between two sets $A,B$ to be

$$d(A,B) = \inf\{\delta : A \subset B_\delta \wedge B \subset A_\delta\}$$

The distance function is known as the Hausdorff metric on $S$. We can also use other distance measures.

Given a LIFS $\{w_1, \ldots. w_m\}$, there exists an unique compact invariant set $F$, such that

$w_i(F)$, this F is known as attractor of the system. If $E$ is compact non-empty subset such that $w_i(E) \subset E$ and

$$w_i(F) \tag{3}$$

We define the k-th iteration of $W$, $W^k(E)$ to be $W^0(E)=E,\ W^k(E)=W(W^{(k-1)}(E))$

For $K \geq 1$ then we have

$$W^k(E) \tag{4}$$

The sequence of iteration $W^k(E)$ converges to the attractor of the system for any set $E$. This means that we can have a family of contractions that approximate complex images and, using the family of contractions, the images can be stored and transmitted in a very efficient way. Once we have a LIFS it is easy to obtain the encoded image.

If we want to encode an arbitrary image in this way, we will have to find a family of contractions so that its attractor is an approximation to the given image. Barnsley's Collage Theorem states how well the attractor of a LIFS can approximate the given image.

(a) Collage Theorem:

Let $\{w_1, \ldots. w_m\}$ be contractions on $R^n$ so that $|w_i(x)-w_i(y)| \leq c|x-y|, \forall\ x,y \in R^n \wedge \forall\ i$,

Where $c<1$. Let $E \subset R^n$ be any non-empty compact set. Then

$$w_i(E))\frac{1}{(1-c)} \tag{5}$$

Where $F$ is the invariant set for the $w_i$ and $d$ is the Hausdorff metric.

As a consequence of this theorem, any subset $R^n$ can be approximated within an arbitrary tolerance by a self-similar set; i.e., given $\delta>0$ there exist contacting similarities $\{w_1, \ldots. w_m\}$ with invariant set $F$ satisfying $d(E,F)<\delta$. Therefore the problem of finding a LIFS $\{w_1, \ldots. w_m\}$ whose attractor $F$ is arbitrary close to a given image $I$ is equivalent to minimizing the distance $w_i(I)$.

### *B. Fractal Image Coding*

The main theory of fractal image coding is based on iterated function system, attractor theorem and Collage theorem. Fractal Image coding makes good use of Image self-similarity in space by ablating image gemetric redundant. Fractal coding process is quite complicated but decoding process is very simple, which makes use of potentials in high compression ratio. Fractal Image coding attempts to find a set of contractive transformations that map (possibly overlapping) domain cells onto a set of range cells that tile the image. One attractive feature of fractal image compression is that it is resolution independent in the sense that when decompressing, it is not necessary that




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the dimensions of the decompressed image be the same as that of original image.

The basic algorithm for fractal encoding is as follows

- The image is partitioned into non overlapping range cells $\{R_i\}$ which may be rectangular or any other shape such as triangles. In this paper rectangular range cells are used.
- The image is covered with a sequence of possibly overlapping domain cells. The domain cells occur in variety of sizes and they may be in large number.
- For each range cell the domain cell and corresponding transformation that best covers the range cell is identified. The transformations are generally the affined transformations. For the best match the transformation parameters such as contrast and brightness are adjusted as shown in Figure 1
- The code for fractal encoded image is a list consisting of information for each range cell which includes the location of range cell, the domain that maps onto that range cell and parameters that describe the transformation mapping the domain onto the range

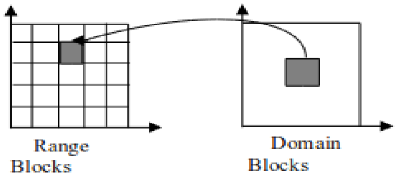

Figure1: Domain-Range Block Transformations

*C. Clustering of an Image*

Clustering of an image can be seen as a starting step to fractal image compression. Long encoding times result from the need to perform a large number of domain-range matches. The total encoding time is the product of the number of matches and the time required for each match. The classification algorithm reduces the encoding time significantly. Classification of domain and ranges is performed to reduce the number of domain-range match computations. Domain-range matches are then performed for those domains that belong to a class similar to the range. The feature computation serves to identify those domains belonging to the class of sub images whose feature vectors are within the feature tolerance value of the feature vector belonging to the range cell. More sophisticated classification schemes use a predefined set of classes. A classifier assigns each domain cell to one of these classes. During encoding, the classifier assigns the range cell to a class and domain range comparisons are performed only against the domains of the same class as the range cell. The classification employed in this work is based on back-propagation algorithm that is trained on feature vector data extracted from domain cells obtained from an arbitrary image.

Two different measures of image variation are used here as features. These particular features were chosen as representative measures of image variation. The specific features used in this work are:

(a) Standard deviation $(\sigma)$ given by

$$\sigma=\sqrt{\frac{1}{n_r n_c}\sum_{i=1}^{n_r}\sum_{j=1}^{n_c}\left(p_{i,j}-\mu\right)^2} \quad (6)$$

Where $\mu$ is the mean or average pixel value over the $n_r \times n_c$ rectangular image segment and $p_{i,j}$ is the pixel value at row $i$, column $j$.

(b) Skewness, which sums the cube of differences between pixel values and the cell mean, normalized by the cube of $\sigma$ is given by

$$\frac{1}{n_r n_c}\sum_{i=1}^{n_r}\sum_{j=1}^{n_c}\frac{\left(p_{i,j}-\mu\right)^3}{\sigma^3} \quad (7)$$

## III. ADAPTIVE NEURO-FUZZY INFERENCE SYSTEM

ANFIS serves as a basis for constructing a set of fuzzy if-then rules with appropriate membership functions to generate the stipulated input-output pairs. Since the main disadvantage of creating a fuzzy system lies in the tuning of the rules, so the concept of neural networks can be incorporated in order to create the rules and membership functions. Generally the fuzzy inference system can be shown as

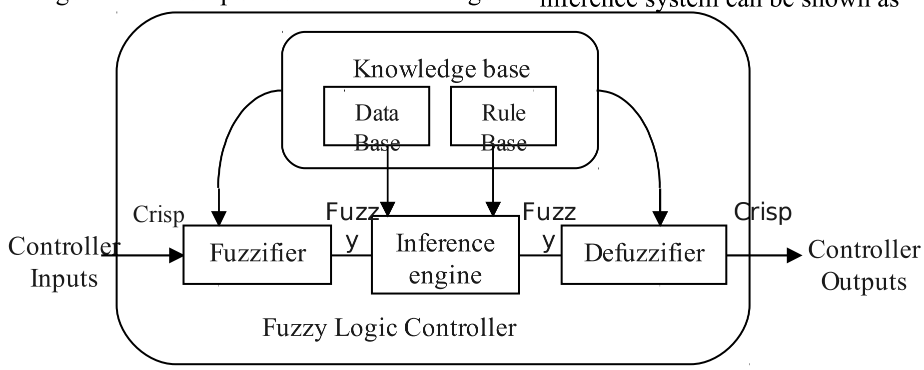


Figure 2: Schematic diagram of Fuzzy building blocks




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The structure of the FLC resembles that of a knowledge-based controller except that the FLC utilizes the principles of fuzzy set theory in its data representation and its logic. The NN structure is designed in such a manner that it trains from the data provided to it. The neural network structure considered for training the fuzzy inference system is shown in figure 3

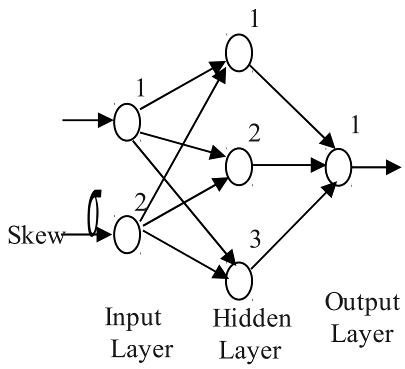


Figure 3: Architecture of NN considered

## IV. Results and Discussions

A gray level image of Lena of size $128 \times 128$ has been considered for training the network using NN and obtaining the structure of ANFIS. A domain pool is created having domains of size $4 \times 4$ for the above image. The standard deviation and skewness for different domains of the above image are calculated using equations (6-7) and the domains are assigned to specific classes based on their values of standard deviation and skewness. The network is trained with the above characteristics of Lena image and the performance is also tested through the ANFIS structure with other gray level image of Barbara of size $256 \times 256$ . The computer simulations have been carried out in MATLAB/SIMULINK environment on Pentium-4 processor with 1.73 GHz and 256 MB RAM and the results have been presented. Figure 4 shows the image of Lena considered for this work.

Figure. 4: Gray level Image of Lena

The data for standard deviation and skewness is collected and their corresponding class of the domain pool is carried out. As a result 3969 data has been collected for this work. Out of this 3000 data has been considered for training of ANFIS and remaining 969 data has been considered for testing of the data. Figure 5 shows the architectural model of ANFIS with two input nodes and one output node considered for this work. Figure 6 and figure 7 show the comparison between the acquired and desired outputs during the testing pattern.

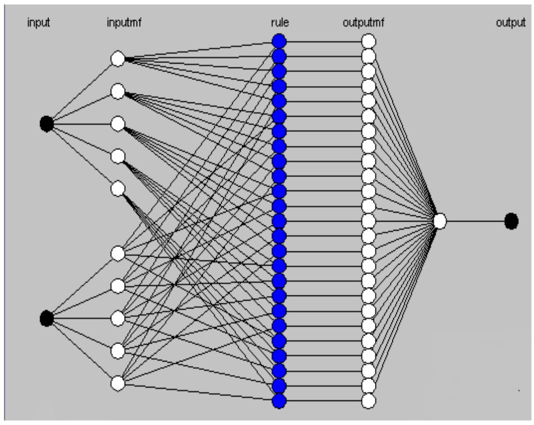


Figure. 5: Architectural model of ANFIS

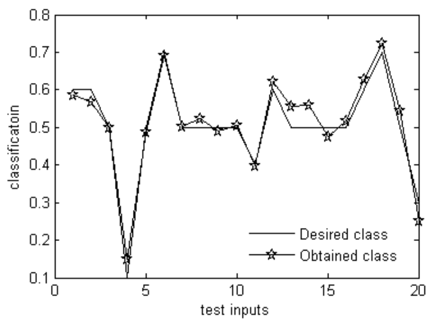


Figure. 6: Comparison between desired and acquired outputs

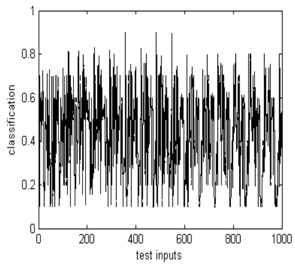


Figure. 7: A clear view of Figure 6

Figure 7 is a clear representation of comparison for the above case by considering a specific number of values. It can be seen from the figures that the desired class and the obtained class are properly superimposed which indicate that the network has been properly trained and it also gives proper output. Figures 8 and 9 show




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the reconstructed images using FIC with ANFIS structure along with the original images of Barbara and Lena. Table 1 shows the performance of ANFIS based FIC in terms of PSNR, Compression ratio and Encoding time over that of FIC using Exhaustive search method [9]. As seen from Table 1 the PSNR value for the gray level image using ANFIS based FIC is less than that of FIC using exhaustive search method. It is due to the reason that the domain-range block comparison is performed only with the domain pool blocks whose classification is same as that of range block which in turn reduces the encoding time. It can also be seen from the table that the encoding time has been greatly reduced by the above proposed technique.

Table 1: Comparison of FIC and ANFIS based FIC

| Image | PSNR (dB) | | Compression ratio (bpp) | | Encoding time (sec) | |
|---|---|---|---|---|---|---|
| | FIC | FIC with ANFIS | FIC | FIC with ANFIS | FIC | FIC with ANFIS |
| Lena | 35.26 | 32.434 | 1.2:1 | 6.73:1 | 8600 | 2350 |
| Barbara | 32.674 | 30.788 | 1.2:1 | 6.73:1 | 8400 | 2209 |

Figure 8a: Original image of Lena

Figure 8b: Reconstructed Image of Lena

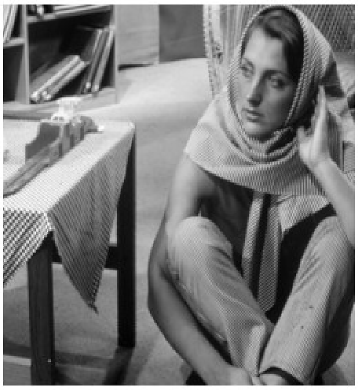

Figure 9a: Original image of Barbara

Figure 9b: Reconstructed Image of Barbara

## V. Conclusions

An ANFIS based network which classifies the domain cells of a gray level image based on its statistical characteristics has been proposed to perform fractal image compression. The conventional methods require a deep understanding of the system dynamics which involves deep mathematical analysis whereas these artificial intelligence techniques can adapt to the system characteristics easily. Computer simulations reveal that performance of ANFIS network




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based fractal image compression is greatly improved in terms of encoding time without degrading the image quality compared to the traditional fractal image compression which employs exhaustive search.

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